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Adap- τ : Adaptively Modulating Embedding Magnitude for Recommendation

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Code: https://github.com/junkangwu/Adap_tau

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- ❑ **Background and Motivation**
- ❑ **Analyses over Embedding Normalization**
- ❑ **Proposed Method**
- ❑ **Experiments**
- ❑ **Summary**

Background and Motivation



- Embedding-based methods achieve competitive performance and support efficient retrieval. **However...**
 - Popular items' embedding **magnitude** grows faster than unpopular ones, which causes excessive contribution to model training and undesirable higher scores due to the potential aggravation of popularity bias from free-varying magnitude.
 - The highly diverse magnitude **prevents model convergence**, even with a proper regularizer, as shown by visual analysis indicating that item embeddings continue to rise instead of converging after numerous epochs.

□ Necessity of Normalization

➤ Theoretical Analysis

LEMMA 1. *By choosing inner product without controlling magnitude, we have change of item embedding magnitude δ_i in each iteration:*

$$\delta_i = \sum_{u \in \mathcal{P}_i} 2\eta \left[\frac{|\mathcal{P}_u| + 1}{m \cdot \mathbb{E}_{j \in I} \exp(\tilde{f}(u, j) - \tilde{f}(u, i))} - 1 \right] \tilde{f}(u, i) \quad (4)$$

At the early stage of the training procedure, δ_i obeys:

$$\delta_i \propto |\mathcal{P}_i| \quad (5)$$

where $\tilde{f}(u, i) = (e_u^T \cdot e_i)$ denotes the inner product of embedding without normalization, $|\mathcal{P}_u|$ and $|\mathcal{P}_i|$ represents the frequency of user u and item i , and \mathcal{P}_i denotes the set of users observed in \mathcal{D} which are interacted with i .

Analyses over Embedding Normalization

□ Necessity of Normalization

➤ Empirical Analysis

- ✓ Free-varying magnitude aggravates popularity bias.

popular items are prone to obtain higher scores as the magnitude directly contributes to model prediction.

- ✓ Free-varying magnitude hurts convergence

The predicted scores and embedding magnitude of unnormalized methods are still in a state of rising rather than convergence while the performance drop consistently

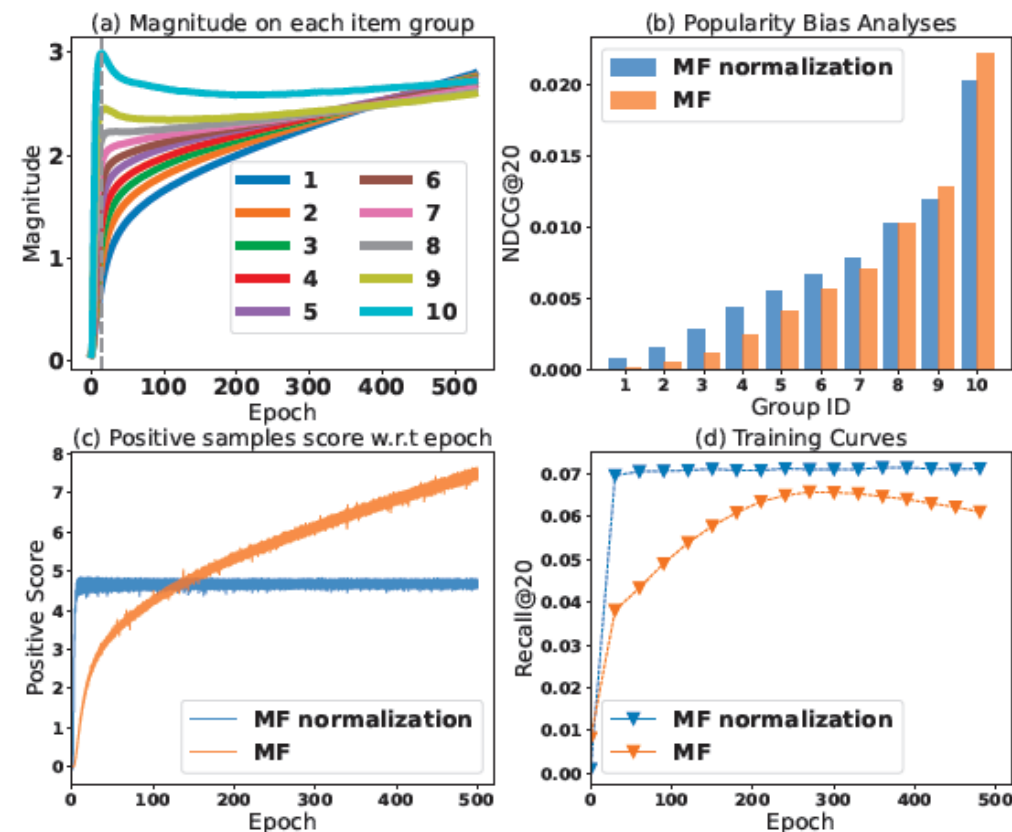


Figure 1: Empirical studies on Yelp2018: Fig. (a) and Fig. (b) represent item embedding magnitude of the different groups across the training procedure and respective performance. The larger GroupID is, the more popular items the group contains. Fig. (c) and Fig. (d) depict the positive samples score and corresponding performance in the training procedure.

Analyses over Embedding Normalization



□ Necessity of Normalization

➤ Empirical Analysis

✓ Normalization boosts performance.

The model with both-side normalization (i.e., Y-Y) remarkably outperforms the model with one-side normalization (i.e., Y-N or N-Y); and they both surpass the model without normalization (N-N).

norm?	Yelp2018		Amazon-book	
	Recall	NDCG	Recall	NDCG
N-N	0.0677	0.0554	0.0457	0.0352
Y-N	0.0709	0.0585	0.0529	0.0419
N-Y	0.0703	0.0577	0.0513	0.0399
Y-Y	0.0714	0.0586	0.0542	0.0422

Analyses over Embedding Normalization



□ Limitation of Normalization

- The performance is highly sensitive to τ
- Different datasets require rather different τ

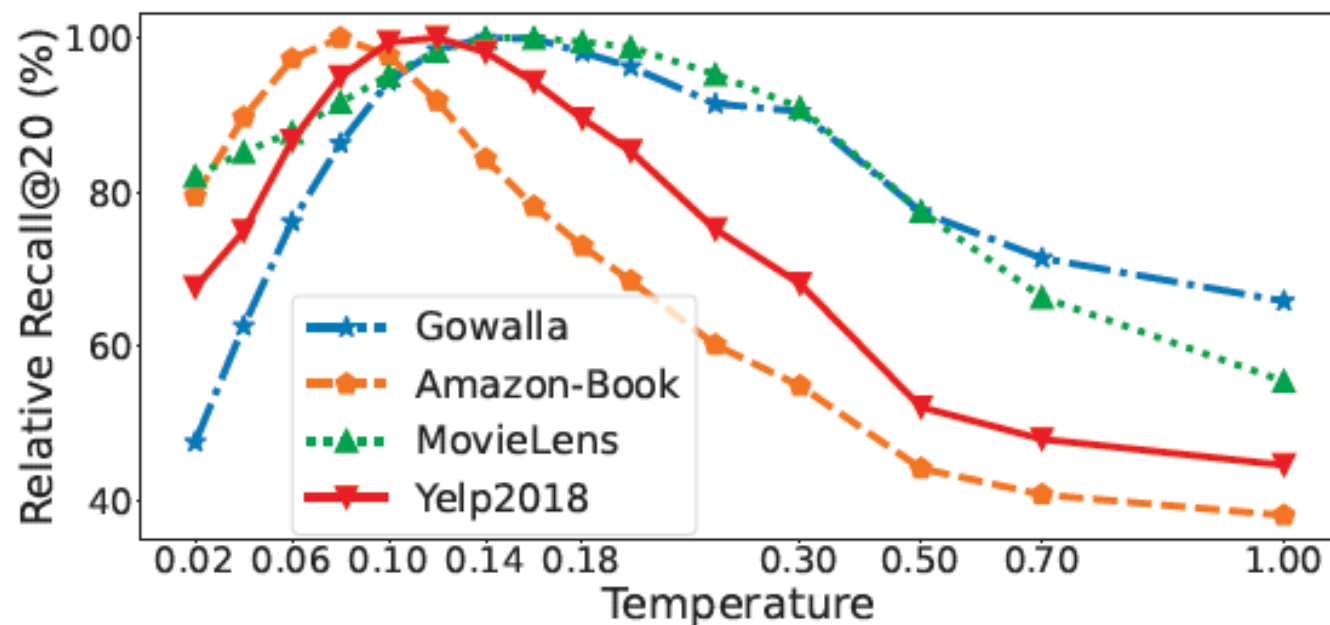


Figure 2: Relative recall@20 over four datasets with τ .

□ Roles of Temperature

➤ Avoiding gradient vanishment.

Too large or too small τ would cause gradient vanishment.

$$\mathbb{E}_i \left[\left| \frac{\partial L}{\partial f(u, i)} \right| \right] = \frac{2}{m\tau} \sum_{i \in \mathcal{P}_u} p_{ui}(\tau) \left(1 - \sum_{k \in \mathcal{P}_u} p_{uk}(\tau) \right)$$

➤ Hard-mining [1].

Too small τ would amplify the disparity and focus on hard negative samples.

- To adaptively and automatically modulate the embedding magnitude, we propose two principles:

(P1) Adaption principle: temperature should be adaptive to avoid gradient vanishing.

(P2) Fine-grained principle: it is beneficial to specify the temperature in a user-wise manner — i.e., the harder the samples of a user are distinguished, the larger temperature should be employed for the user.

□ Adap- τ_0 : Towards Adaptive Temperature

LEMMA 3. Let \mathbb{F} (or \mathbb{F}^+) be the distribution of $f(u, i)$ over all instances (or positive instances). Let f (or f^+) be a random variable that sampled from \mathbb{F} (or \mathbb{F}^+). Suppose the distribution \mathbb{F} and \mathbb{F}^+ have a sub-exponential tail such that the following conditions hold for some

$\lambda, \lambda_+ > 0$:

$$\begin{aligned} p((f - \mathbb{E}_{\mathbb{F}}[f]) > b) &\leq 2e^{-2b/\lambda} \\ p((f_+ - \mathbb{E}_{\mathbb{F}^+}[f_+]) > b) &\leq 2e^{-2b/\lambda_+} \end{aligned} \quad (12)$$

When $\tau_0 \geq \max(2\lambda, 2\lambda_+, T)$, it can be approximated as:

$$\tau_0 \approx \frac{\sigma_+^2 - \sigma^2}{-(\mu^+ - \mu) + \sqrt{(\mu^+ - \mu)^2 + 2(\sigma_+^2 - \sigma^2) \log(\frac{nm}{2|D|})}} \quad (13)$$

where $|D|$ denotes the number of positive instances in the datasets, μ (or μ_+) and σ^2 (or σ_+^2) denotes the mean and variance of f (or f_+). when σ_+^2 is close to σ^2 (cf. Appendix C.1), the expression can be simplified as:

$$\tau_0 \approx \frac{\mu_+ - \mu}{\log(\frac{nm}{2|D|})} \quad (14)$$

□ Adap- τ : Towards Adaptive Fine-grained Temperature

We introduce personalized temperatures τ_u for each user and leverage a Superloss [2] to supervise their learning.

$$J = \frac{L(u) - m_u}{\tau_u} + \beta(\log \tau_u - \log \tau_0)^2$$

In fact, we have a closed-form solution

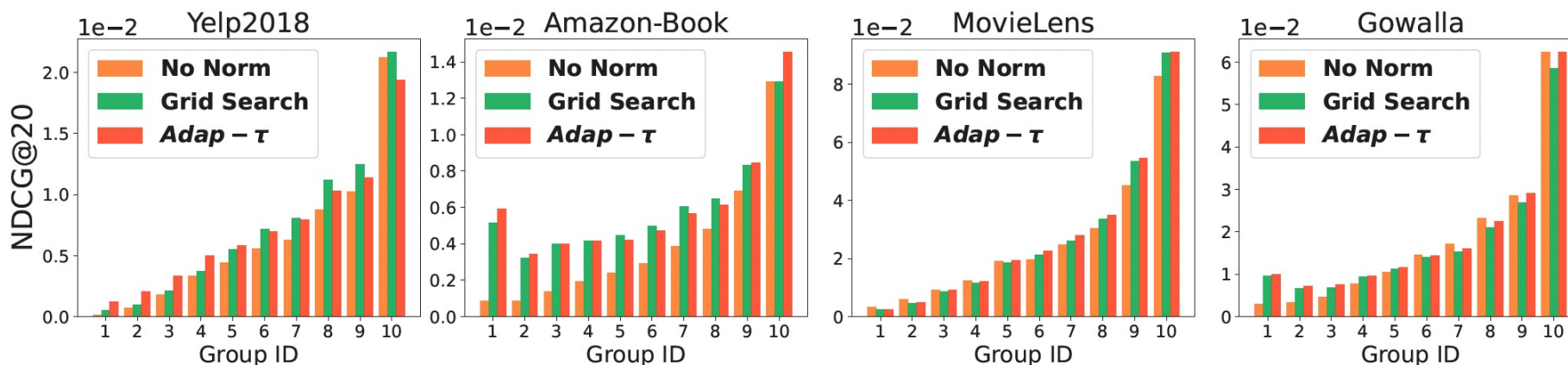
$$\tau_u^* = \tau_0 \cdot \exp(\mathbb{W}(\max(-\frac{1}{e}, \frac{L(u) - m_u}{2\beta})))$$

- ❑ How does Adap- τ perform compared with other strategies?
- ❑ Does our Adap- τ adapt to different datasets and users?
- ❑ How does the model equipped with embedding normalization and adaptive τ perform compared with state-of-the-art in terms of both accuracy and efficiency?

Experiments



Backbone	strategy	Yelp2018		Amazon-book		Movielens		Gowalla	
		Recall	NDCG	Recall	NDCG	Recall	NDCG	Recall	NDCG
MF	No Norm	0.0677	0.0554	0.0457	0.0352	0.2721	0.2525	0.1616	0.1366
	Grid Search τ	0.0714	0.0586	0.0542	0.0422	0.2789	0.2624	0.1761	0.1399
	C- τ	0.0647	0.0528	0.0538	0.0418	0.2472	0.2260	0.1723	0.1362
	Cu- τ	0.0691	0.0566	0.0541	0.0421	0.2600	0.2398	0.1751	0.1383
	Adap- τ_0	0.0714	0.0585	0.0549	0.0427	0.2792	0.2638	0.1754	0.1386
	Adap- τ	0.0721	0.0594	0.0553	0.0430	0.2815	0.2673	0.1838	0.1506
LightGCN	No Norm	0.0649	0.0530	0.0411	0.0315	0.2576	0.2427	0.1830	0.1554
	Grid Search τ	0.0730	0.0605	0.0596	0.0477	0.2767	0.2575	0.1878	0.1577
	C- τ	0.0653	0.0537	0.0571	0.0453	0.2529	0.2282	0.1731	0.1431
	Cu- τ	0.0690	0.0571	0.0586	0.0468	0.2582	0.2357	0.1797	0.1488
	Adap- τ_0	0.0724	0.0603	0.0601	0.0480	0.2744	0.2571	0.1841	0.1526
	Adap- τ	0.0733	0.0612	0.0612	0.0490	0.2787	0.2615	0.1901	0.1590



Experiments

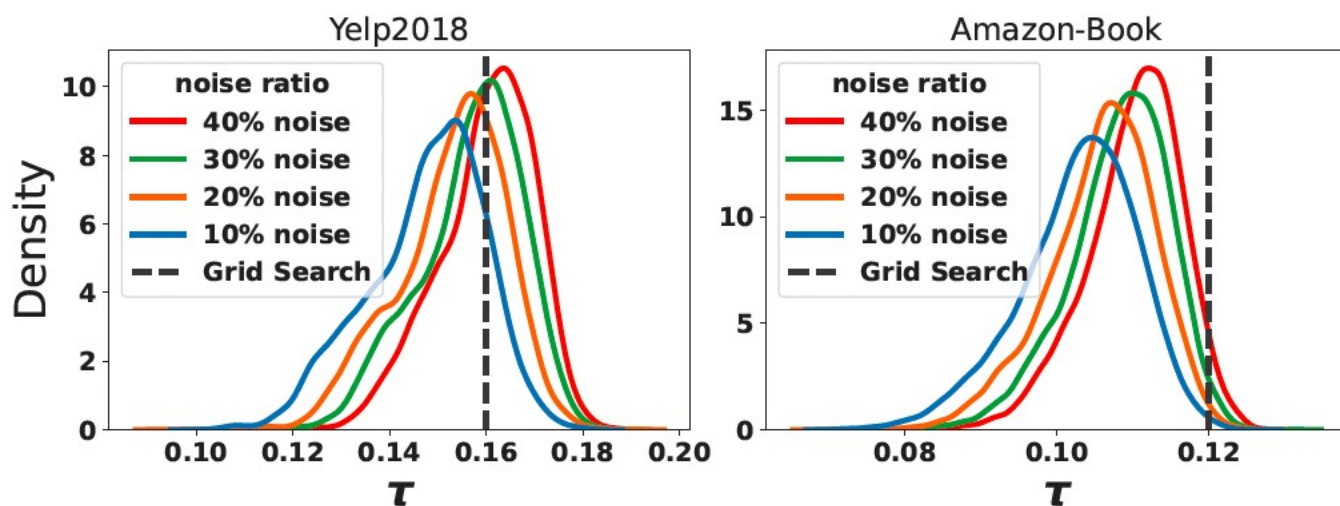
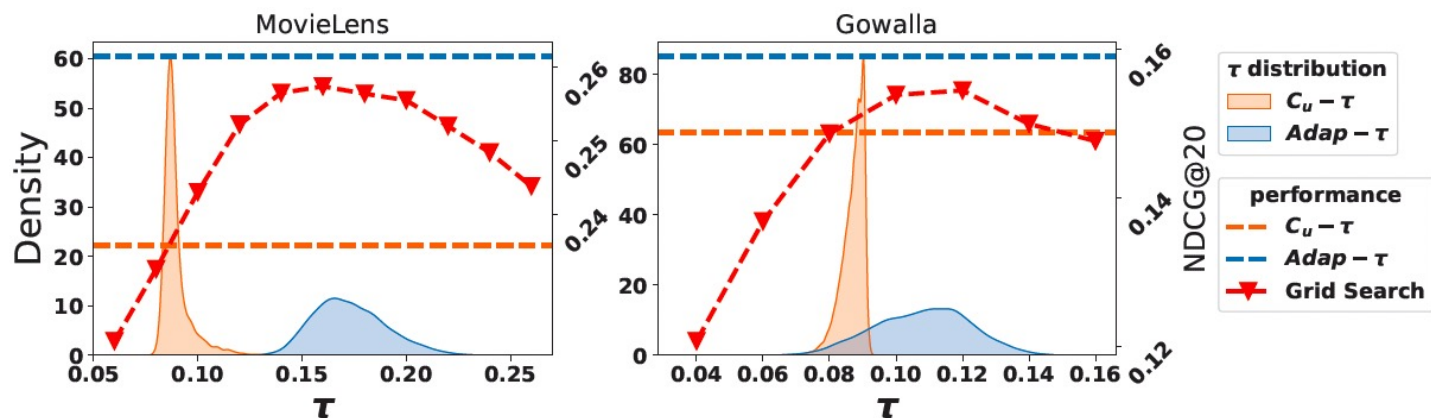


Table 3: Results of MF under different ratio "noisy data"

ratio	model	Yelp2018		Amazon-book	
		Recall	NDCG	Recall	NDCG
0.1	Grid Search	0.0722	0.0601	0.0564	0.0455
	$Adap - \tau$	0.0735	0.0613	0.0577	0.0467
0.2	Grid Search	0.0703	0.0584	0.0534	0.0432
	$Adap - \tau$	0.0717	0.0593	0.0546	0.0443
0.3	Grid Search	0.0696	0.0577	0.0509	0.0409
	$Adap - \tau$	0.0702	0.0584	0.0520	0.0422
0.4	Grid Search	0.0678	0.0563	0.0493	0.0400
	$Adap - \tau$	0.0685	0.0569	0.0507	0.0412
0.5	Grid Search	0.0667	0.0554	0.0481	0.0388
	$Adap - \tau$	0.0672	0.0560	0.0487	0.0394

Experiments

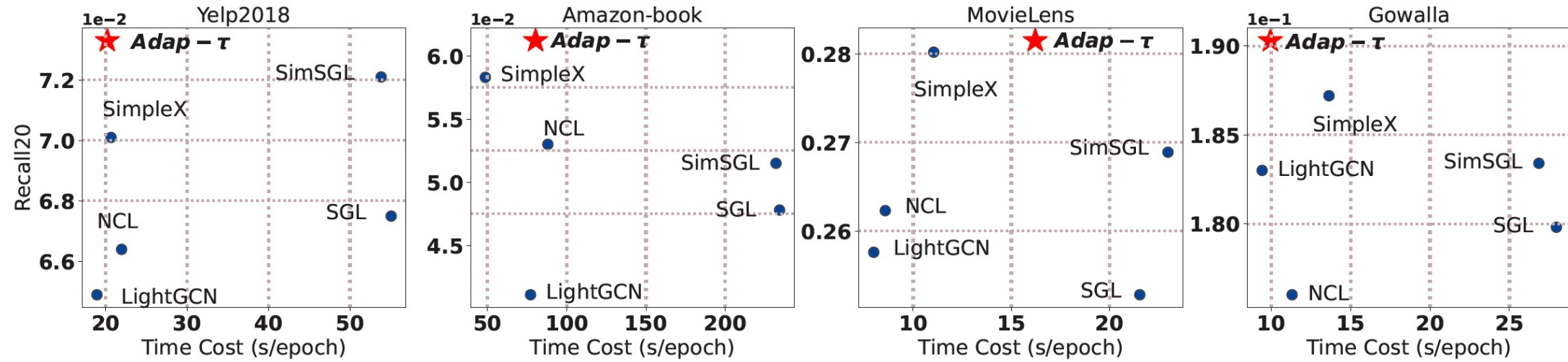


Figure 5: Performance comparisons in terms of both recommendation accuracy and efficiency.

- Embedding normalization is crucial in RS
 - We verify it from theoretical and empirical analysis
 - High sensitive to the Temperature limits its potential

- We provide two principles to guide the adaptive learning of τ
 - We verify it with different backbones in numerous dataset